

Editorial

The present volume of the *Annales Universitatis Paedagogicae Cracoviensis. Studia Mathematica* is devoted to the Lefschetz properties; a topic that continues to connect deep ideas from algebraic geometry, commutative algebra, topology, and combinatorics. It brings together three contributions by leading experts in the field: *Lefschetz Properties through a Topological Lens* by Alexandra Seceleanu, *Lefschetz Properties in Algebra, Geometry and Combinatorics* by Juan C. Migliore and Giuseppe Favacchio, and *Notes on Jordan Type of an Artinian Algebra* by Pedro Macias Marques.

These texts were originally prepared as lecture notes for the *Lefschetz Preparatory School*, held in Kraków on May 6-10, 2024. The School, together with the international conference "Lefschetz Properties in Algebra, Geometry, Topology and Combinatorics", formed the core of the project "Lefschetz Type Properties in Geometry and Algebra" (LEGAL), carried out at by the Department of Mathematics of the University of the National Education Commission in Kraków. Both events were organized with the generous support of the Polish National Agency for Academic Exchange (NAWA) under the *Welcome to Poland programme*, grant no. PPI/WTP/2022/1/00063.

The Preparatory School was designed to introduce young researchers to the vibrant and multifaceted world of Lefschetz-type phenomena, linking classical theorems with modern developments in commutative algebra and combinatorial geometry. It provided an environment where early-career mathematicians could interact closely with internationally renowned specialists, discuss open problems, and experience the collaborative spirit that has long characterized this community. The exceptional didactic quality of the materials prepared for the School, together with the generosity of the lecturers, significantly strengthened this effort.

One of the most tangible outcomes of this initiative has been the visible increase in the participation of young mathematicians in the international Lefschetz community. This was clearly evidenced by their active presence during the *2025 Lefschetz event at the Sophus Lie Center in Nordfjordeid, Norway*, where many of the School's participants contributed to discussions and research sessions. The sustained engagement of this new generation of researchers ensures that the broadly understood Lefschetz programme will continue to evolve and expand across disciplines.

The editors express their gratitude to all the authors for their contributions and to the NAWA programme for its vision and support in strengthening international collaboration in mathematics. We also acknowledge the efforts of the organizing team at the Department of Mathematics of the University of the National Education Commission, whose dedication made both the School and the subsequent conference successful.

It is our hope that this volume will serve both as a lasting record of the lectures delivered in Kraków and as an invitation for further exploration of the fascinating landscape of Lefschetz properties and their many manifestations.